A CONSISTENT SCENARIO FOR $B \to PS$ DECAYS

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We consider $B \to PS$ decays where P stands for pseudoscalar and S for a heavy (1500 MeV) scalar meson. We achieve agreement with available experimental data – which includes a two orders of magnitude hierarchy – assuming the scalars mesons are two quark states. The contribution of the dipolar penguin operator $\mathcal{O}_{11}$ is quantified.

I. INTRODUCTION

The scalar sector below two GeV is poorly understood, nevertheless several features –like the presence of two multiplets and several of their properties – naturally arise in the analysis of a number of authors. A first set of scalars with masses around 1.5 GeV [1] are grouped in a heavy multiplet, including the $K_0^*(1430)$, $a_0(1450)$, $f_0(1500)$ for the octet, $f_0(1370)$ which is identified with the singlet and the $f_0(1710)$ which seems to be mainly glueball. The octet is nearly degenerate, like similar pseudoscalar, vector, axial vector and tensor multiplets, their widths are small (≤ 100 MeV). The mixing angles seems to be small except by the singlet-glueball which is around -20^o, according to H. Y. Cheng in ref. [2]. It has been more difficult to establish the lighter multiplet, even the existence and nature of some of their members is in doubt. The light multiplet should include the $a_0(980)$, $f_0(980)$ and the $\kappa = K_0^*(800)$ in the octet; while the singlet could be identified with the $\sigma = f_0(600)$. The mixing is not clear and their widths are very large. Ideally, the former multiplet can be identified as the ground state of quark antiquark bound states with angular momenta one while the later with the ground state of four quarks systems with zero angular momenta. In the real world an undetermined mixing between the two multiplets is expected. Alternatively both multiplets could be identified as quark-antiquark states with angular momenta one, the lighter being the ground state while the heavier the first excited state.

The full understanding of the scalar multiplets previously described remain a challenge, both from the experimental perspective as well as from the theoretical point of view [1]. To start with, there is not enough and conclusive experimental information regarding the existence and properties of the scalars. Notice that the information is poor not because of the lack of sources of scalar mesons, for example many of the decays of particles containing c or b quarks involves the production of scalar mesons. The information on the scalars is scarce because of the large width they have since that produces a large overlap with nearby resonances and with the background. In spite of those problems, precise experimental results are available [1, 3, 4] for the mass and width of the f_0 and K_0^*, for the β angle [5] of the CKM matrix and for several partial widths. It has been speculated that the α angle can be extracted in processes involving scalars [6] and new projects like the LHCB

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[7] will improve the old measurements and obtain new results. Relevant to our work are the branching ratios for the $B \to PS$ decays measured by different groups, which show a non trivial hierarchy. The experimental data collected in Table I suggest that, for $B \to PS$ decays including members of the heavy scalar multiplet, the order of magnitude of the branching ratios involving the $K_0^*(1430)$, the $f_0(1370)$, $f_0(1500)$ and the $a_0(1450)$ are different.

On the theoretical side the situation is not better. The origin of the difficulties are the non perturbative regime of QCD and the limited computer capacity for the lattice approach. The nature of the observed scalars has been discussed at length and proposals exist to identify them as 2 or 4 quark states, glueballs, molecules, etc. and several theoretical formalisms have been developed to calculate non leptonic decays. The simplest one is the so called 'Naive Factorization Approach' (NFA) [8], which in general produces the correct order of magnitude and its predictions are in rough agreement with the experimental results. Discrepancies are known to occur in two cases, for 'color suppressed' processes and when important re-scattering effects are involved, for example processes where direct CP violation is relevant [9, 10]. The advantage of formalism where a systematic expansion is implemented and where higher order correction can be organized and controlled are of great importance (QCDF, SCET, pQCD, LCSR, etc. [11, 12, 13, 14]), in particular when high accuracy predictions are required.

Additional reasons to study the $B \to PS$ decays are: they offer a window to study the spectroscopy and the dynamics of the scalar sector, the $B \to 3P$ decays get a contribution from the $B \to PS,\ PV,\ PT$, so that in order to achieve an appropriated estimate for the former decay the latter must be well known [15]. In a similar way one can argue that in order to extract signals of possible new physics, the contribution of low lying conventional physics has to be known in detail, including the contributions of the scalar mesons [16]. We believe that the understanding of the physical origin of the hierarchy of scales appearing in the $B \to PS$ decays can shed some light on the nature of the scalars [17, 18]. Complementary information on the nature of the scalars may be obtained from $D \to PS$ physics [19]: in the first case through the decay constants, $\bar{f}^s$ while in the latter through the F^{DS} form factors. The purpose of the present work is to consider the $B \to PS$ decays with S a member of the heavy scalar multiplet. We assume that the leading contribution to these processes is given by the NFA and that, in first approximation, contributions other than the leading one can be safely neglected. In these conditions the dominant contribution can be clearly identified and the existence of the scales in the branching ratios naturally arises. Besides the NFA our approach can be summarized along the following lines: we include ten dimension six four quark operators and the dimension five chromomagnetic operator O_{11} [20], annihilation contributions are included and the form factors required are obtained by using sum rules, so infrared divergences are absent. This approach, together with $SU(3)$ symmetry, allows us to reproduce the pattern observed experimentally.

II. BRANCHING RATIOS AND MIXING

Our results are summarized in Table I. It is worth remarking that both the experimental data and our results points to the existence of branching ratios that ranges from 45 to 0.5 (in units of 10^{-6}). In the following paragraphs we introduce the notation, conventions and explain the procedure we follow to obtain these branching ratios. Within the NFA the hadronic matrix elements can be reduced to products of decay constants and form factors. In order to achieve this one uses the 'vacuum saturation' approximation and neglect other intermediate states. This seems to be a reasonable assumption since the hadronic resonances have masses in the $1-2$ GeV range, far from the m_b region. For the invariant amplitude we write $\mathcal{M}_{f\to i} = \ < f|H|i > \ = \ G_F A_{f\to i}/\sqrt{2}$ while the branching ratios are given by $B = \tau_B G_F^2 |A|^2 p/16\pi m_B^2 = \tau_B G_F^2 |A|^2/32\pi m_B$, with τ_B the B lifetime. The decay constants and form factors are defined as [8, 17, 18]:

$$\langle P(p)|A_\mu|0\rangle \ = \ -if_P p_\mu; \qquad \langle S(p)|V_\mu|0\rangle = f_S p_\mu = \frac{m_2 - m_1}{m_S}\bar{f}_S p_\mu \qquad \langle f_0|q\bar{q}|0\rangle = m_{f_0}\bar{f}_{f_0},$$
$$\langle S(p_2)|L_\mu|P(p_1)\rangle \ = \ -i\left[\left(p_1 + p_2 - \frac{m_1^2 - m_2^2}{q^2}q\right)_\mu F_+^{M_1 M_2} + \frac{m_1^2 - m_2^2}{q^2}q_\mu F_0^{M_1 M_2}(q^2)\right] \qquad (1)$$

Decay	BELLE	BABAR	HFAG [3]	$B_{\text{exp.}}$	NFA	NFA+$\mathcal{O}_{11}$	QCDF [17]	pQCD [17]
$\pi^- a_0^+(1450)(\pi\eta)$		$< 2.3^*$	$< 2.3^*$		8		3.1	
$\pi^+ a_0^-(1450)$					2		0.5	
$\pi^- a_0^0(1450)$					4		2.5	
$\pi^- f_0(1370)$		< 3	< 3					
$\pi^- f_0(1500)$								
$\pi^0 a_0^-(1450)$					0.01		1.1	
$K^+ a_0^-(1450)$	$< 3.1^*$		$< 3.1^*$		1		0.3	
$K^+ a_0^0(1450)$					0.5		0.2	
$K^- f_0(1370)(\pi\pi)$		$< 10.7^*$	$< 10.7^*$	< 41	8	7		
$K^- f_0(1500)(\pi\pi)$		$0.73 \pm 0.21 \pm 0.47^*$	$0.7(5)^*$	2(1)	23	21		55
$\bar{K}^0 a_0^-(1450)$							0.1	
$K^0 a_0^0(1450)$							0.1	
$K^0 f_0(1370)$					7	7		
$K^0 f_0(1500)$					22	21		42
$\pi^- K_0^{*+}(K^+\pi^0)$	$49.7 \pm 3.8 \pm 3.8^{+1.2}_{-4.8}$	$25.4^{+3.0+6.1}_{-3.7-5.6}$	34(5)	34(5)	45	45	11	43
$\pi^+ K_0^{*0}(K^+\pi^-)$	$51.6 \pm 1.7 \pm 6.8^{+1.8}_{-3.1}$	$32.2 \pm 1.2^{10.8}_{-6}$	45(6)	45(6)	45 (in)	45 (in)	11	48
$\pi^0 K_0^{*+}$					25	25	5.3	29
ηK_0^{*+}		$15.8 \pm 2.2 \pm 1.4 \pm 1.7$	16(3)	16(3)	7	7		
$\pi^0 K_0^{*0}$		$11.7^{+1.4+4}_{-1.3-3.6}$	12(4)	12(4)	17	17	6.4	18
ηK_0^{*0}		$9.6 \pm 1.4 \pm 0.7 \pm 1.1$	10(2)	10(2)	7	7		

TABLE I: Branching ratios for the $B \to PS$ decays (in units of 10^{-6}), for the heavier scalar multiplet. The values reported for the widths marked with * include the corresponding branching of the scalar decaying channel. To obtain the NFA predictions we used $B(f_0(1370) \to 2\pi) = 0.26(1)$, $B(f_0(1500) \to 2\pi) = 0.35(2)$ and for the $a_0(1450) \to \pi\eta$ no reliable value exists[3].

with $q = p_1 - p_2$.

We have left to the appendix details regarding the effective Hamiltonian we use - which includes ten dimension six operators and the so called $\mathcal{O}_{11}$ operator - and the matrix elements evaluation. The most interesting decays are those involving the $S = K_0^*(1430)$ both because they have the largest branching ratio (around 40, in units of 10^{-6}) and because the theoretical predictions are the cleanest. The a_6 term is by far the dominant one. The amplitudes are proportional to $\lambda_{ts} f_{K_0^*} a_6 m_{K_0^*}^2 / m_s m_b \sim \lambda_{ts} a_6 m_b m_{K_0^*} \bar{f}_{K_0^*}$ times $SU(3)$ factors. The origin of the enhancement is a combination of large CKM matrix elements, the novanishing decay constant and a large $m_{K_0^*}$ (Chiral enhancement) mass. The $SU(3)$ symmetry allow us to relate different decays involving the K_0^* and so, by measuring one of them, one can predict the others, a fact that is not distorted by the $\mathcal{O}_{11}$ contributions. For the numerical analysis we used the following input parameters: $F^{B\pi} = 0.27(4)$, $F^{BK} = 0.33(4)$, $m_s(2.1) = 90$ MeV, $F^{Ba_0(1450)} = F^{BK_0^*(1430)} = 0.26$ and, when required, $SU(3)$ relations are invoked. Although predictions for $f_{K_0^*}$ are available [17], we prefered to include the $B^+ \to \pi^+ K_0^{*0}$ experimental value as an input, obtaining thus $f_{K_0^*}^{\text{eff.}} \simeq 58$ MeV ($f_{K_0^*}^{\text{eff.}} \simeq 56$ MeV when the $\mathcal{O}_{11}$ is taken into account). The branching ratios we obtain for other channels involving the K_0^* are reported in Table I. Notice that the value obtained for $f_{K_0^*}$ is not far from the theoretical predictions (see Table II).

We now consider the decays involving $S = f_0(1370)$, $f_0(1500)$ and $f_0(1700)$. Their relevance stem from the large branching ratios predicted for them [17] – of the same order as the K_0^* – and also due to the possible glueball nature of the $f_0(1700)$. Their amplitudes are proportional to $\lambda_{ts} a_6 m_b m_{K_0^*} \bar{f}_{f_0}^s$ times $SU(3)$ factors and mixing angles (s- quark content). Our predictions for these processes are included in Table I, unfortunately the experimental results are still inconclusive. Note that except the $f_0(1500)$ decay channel, the NFA plus $SU(3)$ symmetry for the heavy scalar multiplet leads predictions for the branching ratios in rough agreement with

the experimental values. However, even if the experimental data is poor the discrepancy between our results and experimental data is evident, there is a one order of magnitude difference. In this sense it is important to remark that in order to obtain the results of tabler I we assumed, following H. Y. Cheng [2] a mixing between the glueball, singlet and octet components given by:

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1700) \end{pmatrix} = \begin{pmatrix} 0.78 & 0.51 & -0.36 \\ -0.54 & 0.84 & 0.03 \\ 0.32 & 0.18 & 0.93 \end{pmatrix} \begin{pmatrix} N \\ S \\ G \end{pmatrix}$$
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -s_{23}c_{12} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} N \\ S \\ G \end{pmatrix} \tag{2}$$

where $s_i = \sin\theta_i$ and so on. The angles $\theta_{12} \simeq 2^o$, $\theta_{13} \simeq -21^o$ and $\theta_{23} \simeq 2^o$ are the mixing between singlet-octet, singlet-glueball and octet-glueball, respectively. The singlet and the octet are $f_0(1370) \sim f_{\rm sing.} = \sqrt{2/3}\ N + S/\sqrt{3}$, $f_0(1500) \sim f_{\rm oct.} = N/\sqrt{3} - S\sqrt{2/3}$, $S = \bar{s}s$, $N = \left(\bar{u}u + \bar{d}d\right)/\sqrt{2}$ and $G = gg$ the glueball. Thus, in this approach [2], there is only an small mixing between the singlet and the glueball. Using these values the prediction for $B \to f_0(1500)K$ is in conflict with the experimental data. One way to avoid this problem is to leave θ_{12} as a free parameter, keeping the others fixed. Using the experimental data we obtain the following inequality for the mixing between the singlet and the octet:

$$|-s_{12}\sqrt{\frac{1}{3}} + c_{12}\sqrt{\frac{2}{3}}| \ \leq\ 0.34. \tag{3}$$

These constraints lead two possible values:

$$35^\circ \leq\ \theta_{12}\ \leq 74^\circ \tag{4}$$
$$215^\circ \leq\ \theta_{12}\ \leq 254^\circ \tag{5}$$

It is worth noticing that these values for the mixing are close to those mentioned by several groups [1].

Finally for the decays involving the $a_0(1450)$, $B \to a_0(1450)\pi, a_0(1450)K$, the terms proportional to $a_4 - a_6 \sim 0$ almost vanish and the branching ratios are smaller. Two different cases must be considered. The first when the amplitude is dominated by the tree level contribution a_1 (The amplitudes are proportional to $\lambda_{ud} a_1 m_B^2 f_\pi$), then the theoretical prediction is reliable and the branchings are predicted to be are around 10 (in units of 10^{-6}). The second case arises when no tree level contribution exist and terms like annihilation are dominant. In this case the branchings are of order 0.1-1 (in units of 10^{-6}) but the theoretical uncertainties are larger since other contributions (FSI for example[10]) maybe important. Unfortunately little is known about these corrections.

III. SUMMARY

In this work we studied the $B \to PS$ decay where S stands for a member of the heavy scalar multiplet. The computation have been done assuming the heavy scalar multiplet is a two quark states, using $SU(3)$ symmetry and the naive factorization approach. Our conclusions can be summarized as follows:

- Within the error bars, it is possible to reproduce the hierarchy of branching ratios experimentally observed in the $B \to PS$ decays, whether or not the operator $\mathcal{O}_{11}$ is included.

Ref.	$(f/\bar{f})_{K_0^*(1430)}, \bar{f}$	$\bar{f}_{a_0(1450)}$	$\bar{f}^s_{f_0(1500)}$	m_s [GeV]
Meurice-87 [21]	27	-	-	
Narison-89 [21]	40(6)	-	-	
Maltman [21]	42(2)	390(159)	-	
Chernyak-01 [21]	70(10)	-	-	
Shakin-01 [21]	30	207	-	
Pennington-01 [21]	-	-	-	
Du-04 [21]	42(8), 427(85)		-	0.14
Cheng-05 [17] at $\mu = 1$ GeV	445(50)	460(50)	490(50)	0.119
Cheng-05 [17] at $\mu = 2.1$ GeV	550(60)	570(60)	605(60)	0.09
lattice-06 [1]				

TABLE II: Decay constants for scalars (in MeV). The heavy scalars are assumed to be two quark states. Notice that the constants computed by Cheng, were obtained by using sum rules, OPE and Renormalization Group equations that render $\bar{f}$ scale dependant.

- When the singlet-octet mixing given by [1] is used, we obtain a prediction for the $f_0(1500)$ which is one order of magnitude above the experimental limit. A solution to this problem can be obtained by modifying the mixing matrix. In such a case one obtain a constrain on the singlet-octet mixing and its s-quark content.
- The contribution of the $\mathcal{O}_{11}$ operator is around 30 % in decay channels involving the K_0^*. The $\mathcal{O}_{11}$ contributions approximately keep the $SU(3)$ relations between different decay channels.
- The chiral enhancement predicted by the NFA could be used to test the quark structure of the heavy multiplet. Strong deviations from the NFA results could be interpreted as a signal that the heavy scalars are not pure two quark state.

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APPENDIX: NAIVE FACTORIZATION APPROACH (NFA)

The relevant effective Hamiltonian is given by [8] :

$$\mathcal{H}_{eff} \;=\; \frac{G_F}{\sqrt{2}} \left[V_{ub} V_{uq}^* \left(C_1 O_1^u + C_2 O_2^u \right) - V_{tb} V_{tq}^* \left(\sum_{i=3}^{10} C_i \, O_i + C_g O_g \right) \right] + \text{h.c.} \tag{A.1}$$

where $\lambda_{q'q} = V_{q'b} V_{q'q}^*$, with $q = d, s$, while $q' = u, c, t$. The Cabibbo-Kobayashi-Maskawa (CKM) matrix elements are denoted by V_{ij}. O_i stand for the following four fermion operators:

$$\begin{aligned}
\mathcal{O}_1 &= (\bar{q}u)_L (\bar{u}b)_L, & \mathcal{O}_2 &= (\bar{u}_\alpha b_\beta)_L (\bar{q}_\beta u_\alpha)_L, \\
\mathcal{O}_3 &= (\bar{q}b)_L \textstyle\sum_{q'} (\bar{q}'q')_L, & \mathcal{O}_4 &= (\bar{q}_\alpha b_\beta)_L \sum_{q'} (\bar{q}'_\beta q'_\alpha)_L, \\
\mathcal{O}_5 &= (\bar{q}b)_L \textstyle\sum_{q'} (\bar{q}'q')_R, & \mathcal{O}_6 &= (\bar{q}_\alpha b_\beta)_L \sum_{q'} (\bar{q}'_\beta q'_\alpha)_R = -2 \sum_{q'} (\bar{q}'b)_{S-P} (\bar{q}q')_{S+P}, \\
\mathcal{O}_7 &= \tfrac{3}{2} (\bar{q}b)_L \textstyle\sum_{q'} e_{q'} (\bar{q}'q')_R, & \mathcal{O}_8 &= \frac{3}{2} (\bar{q}_\alpha b_\beta)_L \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_R = -3 \sum_{q'} e_{q'} (\bar{q}'b)_{S-P} (\bar{q}q')_{S+P}, \\
\mathcal{O}_9 &= \tfrac{3}{2} (\bar{q}b)_L \textstyle\sum_{q'} e_{q'} (\bar{q}'q')_L, & \mathcal{O}_{10} &= \frac{3}{2} (\bar{q}_\alpha b_\beta)_L \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_L = \frac{3}{2} \sum_{q'} e_{q'} (\bar{q}'b)_L (\bar{q}q')_L.
\end{aligned} \tag{A.2}$$

and the dipole penguin operator:

$$\mathcal{O}_{11} = \frac{g_s}{16\pi^2} m_b \bar{q} \sigma_{\mu\nu} R T_a b G_a^{\mu\nu};$$

with $T_a = \lambda_a/2$ the $SU(3)_C$ generators. The Wilson coefficients C_i appear in the combinations $a_{2i-1} = C_{2i-1} + C_{2i}/N$, $a_{2i} = C_{2i} + C_{2i-1}/N$. The numerical values are taken from [8]. Similarly we define $a_{11} = (8/9)\alpha_s C_{11} (m_b^2/4\pi q^2) \simeq -5.7 \cdot 10^{-3}$. For the gluon momentum we use $q \simeq p_b - p_s \simeq p_B - p_k/2$, so $q^2 \simeq m_B^2/2$ [20]. Taking $\alpha_s(q^2 \simeq m_B^2/2) = 0.21$ and $C_{11} = -0.29$ one obtains $a_{11} \simeq -5.7 \cdot 10^{-3}$. Chiral projections are $L,\ R = 1 \mp \gamma_5$. Using the relation $2(T_i)_{\alpha\beta}(T_i)_{\gamma\delta} = \delta_{\alpha\delta}\delta_{\beta\gamma} - (1/N)\delta_{\alpha\gamma}\delta_{\beta\delta}$ and the Fiertz reordering one obtains for $\mathcal{O}_{11}$ [20]

$$\begin{aligned}
\mathcal{H}_{11} \;=\; & i \frac{G_F}{\sqrt{2}} \lambda_{tq} \frac{C_g \alpha_S m_b}{8\pi k^2} \left[\frac{N_C^2 - 1}{N_C^2} \delta_{\alpha\beta}\delta_{\gamma\delta} - \frac{2}{N_C} T^a_{\alpha\beta} T^a_{\gamma\delta} \right] \\
& k_\mu \left[3i \bar{q}_\alpha R \gamma^\mu q'_\beta \bar{q}'_\gamma R b_\delta - 3i \bar{q}_\alpha R q'_\beta \bar{q}'_\gamma \gamma^\mu R b_\delta + \bar{q}_\alpha R \gamma_\nu q'_\beta \bar{q}'_\gamma \sigma^{\nu\mu} R b_\delta - \bar{q}_\alpha R \sigma^{\mu\nu} q'_\beta \bar{q}'_\gamma \gamma_\nu R b_\delta \right]
\end{aligned} \tag{A.3}$$

where $k^2 \simeq m_B/2 - m_K^2/8$.
The amplitudes, including O_{11} contribution, in the NFA are given by:

$$\begin{aligned}
A_{\bar{B}^0 \to \pi^- a_0^+} \;\simeq\; & \lambda_{ud} (a_1 X^{\pi^-}_{\bar{B}^0 a_0^+} + a_2 X^{\bar{B}^0}_{(a_0^+ \pi^-)_u}) - \lambda_{td} \Bigg[\left(a_4 + a_{10} - \frac{(a_6 + a_8) m_\pi^2}{\hat{m}(m_b + m_u)} \right) X^{\pi^-}_{\bar{B}^0 a_0^+} \\
& + \left(2(a_3 - a_5) + a_4 + \frac{a_9 - a_7 - a_{10}}{2} - \frac{(a_6 - a_8/2) m_B^2}{m_u(m_b + m_d)} \right) X^{\bar{B}^0}_{(a_0^+ \pi^-)_u} \Bigg]
\end{aligned}$$

$$A_{\bar{B}^0\to\pi^+a_0^-} \simeq \lambda_{ud}(a_1 X^{a_0^-}_{\bar{B}^0\pi^+} + a_2 X^{\bar{B}^0}_{(a_0^-\pi^+)_u}) - \lambda_{td}\Bigg[(a_4+a_{10})X^{a_0^-}_{\bar{B}^0\pi^+} - 2(a_6+a_8)\tilde{X}^{a_0^-}_{\bar{B}^0\pi^+}$$
$$+ \left(2(a_3-a_5)+a_4+\frac{a_9-a_7-a_{10}}{2} - \frac{(a_6-a_8/2)m_B^2}{m_u(m_b+m_d)}\right) X^{\bar{B}^0}_{(a_0^-\pi^+)_u}\Bigg]$$

$$A_{B^-\to\pi^-S^0} \simeq \lambda_{ud}a_1(X^{\pi^-}_{B^-S^0} + X^{B^-}_{S^0\pi^-}) - \lambda_{td}\Bigg[\left(a_4+a_{10}-\frac{(a_6+a_8)m_\pi^2}{\hat{m}(m_b+m_u)}\right) X^{\pi^-}_{B^-S^0}$$
$$+ \left(a_4+a_{10}-\frac{(a_6+a_8)m_B^2}{\hat{m}(m_b+m_u)}\right) X^{B^-}_{S^0\pi^-} + (a_8-2a_6)\tilde{X}^{S_d^0}_{B^-\pi^-}\Bigg]$$

$$A_{B^-\to\pi^0a_0^-} \simeq \lambda_{ud}\left[a_1(X^{a_0^-}_{B^-\pi^0} + X^{B^-}_{a_0^-\pi^0}) + a_2 X^{\pi_u^0}_{B^-a_0^-}\right] - \lambda_{td}\Bigg[(a_4+a_{10})\, X^{a_0^-}_{B^-\pi^0} - 2(a_6+a_8)\tilde{X}^{a_0^-}_{B^-\pi^0}$$
$$- \left(a_4 - \frac{3}{2}(a_9-a_7) - \frac{1}{2}a_{10} - \frac{(a_6+a_8)m_\pi^2}{m_u(m_b+m_d)}\right) X^{\pi_u^0}_{B^-a_0^-} + \left(a_4+a_{10}-\frac{(a_6+a_8)m_B^2}{\hat{m}(m_b+m_u)}\right) X^{B^-}_{a_0^-\pi^0}\Bigg]$$

$$A_{\bar{B}^0\to\pi^0S^0} \simeq \lambda_{ud}a_2(X^{\pi_u^0}_{\bar{B}^0a_0^0} + X^{\bar{B}^0}_{(a_0^0\pi^0)_u}) - \lambda_{td}\Bigg[\left(-\frac{3}{2}a_7 + \frac{(2a_6-a_8)m_\pi^2}{2m_d(m_b-m_d)}\right) X^{\pi_u^0}_{\bar{B}^0a_0^0}$$
$$+ \left(2a_5 + \frac{3}{2}a_7 + \frac{(2a_6-a_8)m_B^2}{2m_d(m_b+m_d)}\right) X^{\bar{B}^0}_{(a_0^0\pi^0)_u}\Bigg]$$

$$A_{\bar{B}^0\to K^-a_0^+} \simeq \lambda_{us}a_1 X^{K^-}_{\bar{B}^0\pi^+} - \lambda_{ts}\Bigg[\left(a_4+a_{10}-(a_6+a_8)r_\chi^K\right) X^{K^-}_{\bar{B}^0a^+}$$
$$+ \left(a_4 - \frac{1}{2}a_{10} - \frac{(2a_6-a_8)m_B^2}{(m_b+m_d)(m_s+m_d)}\right) X^{\bar{B}^0}_{K^-a_0^+}\Bigg]$$

$$A_{B^-\to K^-S^0} \simeq \lambda_{us}a_1\left[X^{K^-}_{B^-S^0} + X^{B^-}_{S^0K^-}\right] - \lambda_{ts}\Bigg[\left(a_4+a_{10}-(a_6^{\text{eff.}}+a_8)r_\chi^K\right) X^{K^-}_{B^-S^0} - \left(2a_6 - a_8 - \frac{30}{32}a_{11}\right)\tilde{X}^{S_s^0}_{B^-K^-}$$
$$+ \left(a_4+a_{10}-\frac{2(a_6+a_8)m_B^2}{(m_b+m_u)(m_s+m_u)}\right) X^{B^-}_{S^0K^-}\Bigg]$$

$$A_{B^-\to\bar{K}^0a_0^-} \simeq \lambda_{us}a_1 X^{B^-}_{\bar{K}^0a_0^-} - \lambda_{ts}\Bigg[\left(a_4 - \frac{1}{2}a_{10} - (a_6-a_8/2)r_\chi^K\right) X^{\bar{K}^0}_{B^-a_0^-}$$
$$+ \left(a_4+a_{10}-\frac{2(a_6+a_8)m_B^2}{(m_b+m_u)(m_s+m_u)}\right) X^{B^-}_{\bar{K}^0a_0^-}\Bigg]$$

$$A_{\bar{B}^0\to\bar{K}^0S^0} \simeq -\lambda_{ts}\Bigg[\left(a_4 - \frac{a_{10}}{2} - (a_6^{\text{eff.}}-a_8/2)r_\chi^K\right) X^{\bar{K}^0}_{\bar{B}^0S^0} - \left(2a_6-a_8-\frac{30}{32}a_{11}\right)\tilde{X}^{S_s^0}_{\bar{B}^0\bar{K}^0}$$
$$+ \left(a_4 - \frac{a_{10}}{2} - \frac{(2a_6-a_8)m_B^2}{(m_b+m_d)(m_s+m_d)}\right) X^{\bar{B}^0}_{S^0\bar{K}^0}\Bigg]$$

$$A_{\bar{B}^0\to\pi^+K_0^{*-}} \simeq \lambda_{us}a_1 X^{K_0^{*-}}_{\bar{B}^0\pi^+} - \lambda_{ts}\Bigg[\left(a_4+a_{10}-(a_6^{\text{eff.}}+a_8)r_\chi^*\right) X^{K_0^{*-}}_{\bar{B}^0\pi^+}$$

$$
\begin{aligned}
&+\left(a_4-\frac{1}{2}a_{10}-\frac{(2a_6-a_8)m_B^2}{(m_b+m_d)(m_s+m_d)}\right)X^{\bar B^0}_{\pi^+K_0^{*-}}\Bigg]\\
A_{B^-\to\pi^-\bar K_0^{*0}} &\simeq \lambda_{us}a_1X^{B^-}_{\bar K_0^{*0}\pi^-}-\lambda_{ts}\Bigg[\left(a_4-\frac{1}{2}a_{10}-(a_6^{\rm eff.}-a_8/2)r_\chi^*\right)X^{\bar K_0^{*0}}_{B^-\pi^-}\\
&+\left(a_4+a_{10}-\frac{2(a_6+a_8)m_B^2}{(m_b+m_u)(m_s+m_u)}\right)X^{B^-}_{\bar K_0^{*0}\pi^-}\Bigg]\\
A_{B^-\to\pi^0K_0^{*-}} &\simeq \lambda_{us}\left[a_1\left(X^{K_0^{*-}}_{B^-\pi^0}+X^{B^-}_{\pi^0K_0^{*-}}\right)+a_2X^{\pi_u^0}_{B^-K_0^{*-}}\right]-\lambda_{ts}\Bigg[\left(a_4+a_{10}-(a_6^{\rm eff.}+a_8)r_\chi^*\right)X^{K_0^{*-}}_{B^-\pi^0}\\
&+\left(a_4+a_{10}-\frac{2m_B^2(a_6+a_8)}{(m_b+m_u)(m_s+m_u)}\right)X^{B^-}_{K_0^{*-}\eta}+\frac{3}{2}(a_9-a_7)X^{\pi_u^0}_{B^-K_0^{*-}}\Bigg]\\
A_{B^-\to\eta K_0^{*-}} &\simeq \lambda_{us}\left[a_1\left(X^{K_0^{*-}}_{B^-\eta}+X^{B^-}_{\eta K_0^{*-}}\right)+a_2X^{\eta_u}_{B^-K_0^{*-}}\right]-\lambda_{ts}\Bigg[\left(a_4+a_{10}-(a_6^{\rm eff.}+a_8)r_\chi^*\right)X^{K_0^{*-}}_{B^-\eta}\\
&+\left(\frac{3}{2}(a_9-a_7)-2a_4+a_{10}+(2a_6-a_8)r_\chi^{\eta_s}+\frac{a_{11}}{64}\left(12-30r_\chi^{\eta_s}-1\right)\right)X^{\eta_u}_{B^-K_0^{*-}}\Bigg]\\
&+\left(a_4+a_{10}-\frac{2m_B^2(a_6+a_8)}{(m_b+m_u)(m_s+m_u)}\right)X^{B^-}_{K_0^{*-}\eta}
\end{aligned}
$$

$$
\begin{aligned}
A_{\bar B^0\to\pi^0\bar K_0^{*0}} &\simeq \lambda_{us}a_2X^{\pi_u^0}_{\bar B^0\bar K_0^{*0}}-\lambda_{ts}\Bigg[\left(a_4-\frac{a_{10}}{2}-(a_6^{\rm eff.}-a_8/8)r_\chi^*\right)X^{\bar K_0^{*0}}_{\bar B^0\pi^0}+\frac{3}{2}(a_9-a_7)X^{\pi_u^0}_{\bar B^0\bar K_0^{*0}}\\
&+\left(a_4-\frac{a_{10}}{2}-\frac{(2a_6-a_8)m_B^2}{(m_b+m_d)(m_s+m_d)}\right)X^{\bar B^0}_{\bar K_0^{*0}\pi^0}\Bigg]\\
A_{\bar B^0\to\eta\bar K_0^{*0}} &\simeq \lambda_{us}a_2X^{\eta_u}_{\bar B^0\bar K_0^{*0}}-\lambda_{ts}\Bigg[\left(a_4-\frac{a_{10}}{2}-(a_6^{\rm eff.}-a_8/2)r_\chi^*\right)X^{\bar K_0^{*0}}_{\bar B^0\eta}\\
&+\left(\frac{3}{2}(a_9-a_7)-2a_4+a_{10}+(2a_6-a_8)r_\chi^{\eta_s}+\frac{a_{11}}{64}\left(12-30r_\chi^{\eta_s}-1\right)\right)X^{\eta_u}_{\bar B^0\bar K_0^{*0}}\\
&+\left(a_4-\frac{a_{10}}{2}-\frac{(2a_6-a_8)m_B^2}{(m_b+m_d)(m_s+m_d)}\right)X^{\bar B^0}_{\bar K_0^{*0}\eta}\Bigg]
\end{aligned}
\qquad\text{(A.4)}
$$

and $A_{\bar B^0\to K_0^{*+}\pi^-}=0$, $S^0=a_0^0$, σ and f_0, $r_\chi^*\simeq 2m^2_{K_0^*}/m_bm_s$, $r_\chi^{\eta_s}\simeq m_\eta^2/m_bm_s$ and $a_6^{\rm eff.}r_\chi^M=a_6r_\chi^M-a_{11}\left[12(1-r_\chi^M)-1\right]/32$.

$$
\begin{aligned}
X^{K^-}_{B^-f_0} &=<K^-|(\bar us)_L|0><f_0|(\bar ub)_L|B^->=f_K(m_B^2-m_{f_0}^2)F_0^{B^-f_0}(m_\pi^2)=f_K\frac{F_0^{\bar B^0a_0^+}(m_K^2)}{\sqrt2}(m_B^2-m_{f_0}^2)\sin\phi_S\\
X^{\bar K^0}_{\bar B^0f_0} &=<\bar K^0|(\bar sd)_L|0><f_0|(\bar db)_L|\bar B^0>=f_{\bar K^0}(m_B^2-m_{f_0}^2)F_0^{\bar B^0f_0}(m_K^2)=f_K\frac{F_0^{\bar B^0a_0^+}(m_K^2)}{\sqrt2}(m_B^2-m_{f_0}^2)\sin\phi_S\\
X^{K_0^{*-}}_{\bar B^0\pi^+} &=<K_0^{*-}|(\bar su)_L|0><\pi^+|(\bar ub)_L|\bar B^0>=f_{K_0^{*-}}(m_B^2-m_\pi^2)F_0^{\bar B^0\pi^+}(m^2_{K_0^*})=-f_{K_0^{*-}}(m_B^2-m_\pi^2)F_0^{B^0\pi^-}(m^2_{K_0^*})\\
X^{\bar K_0^{*0}}_{B^-\pi^-} &=<\bar K_0^{*0}|(\bar sd)_L|0><\pi^-|(\bar db)_L|B^->=f_{\bar K_0^{*0}}(m_B^2-m_\pi^2)F_0^{\bar B^-\pi^-}(m^2_{K_0^*})=-f_{K_0^{*0}}(m_B^2-m_\pi^2)F_0^{B^0\pi^-}(m^2_{K_0^*})\\
X^{K_0^{*-}}_{B^-\pi^0} &=<K_0^{*-}|(\bar su)_L|0><\pi_0|(\bar ub)_L|B^->=f_{K_0^{*-}}(m_B^2-m_{\pi^0}^2)F_0^{B^-\pi_0}(m^2_{K_0^*})=-f_{K_0^*}\frac{F_0^{B^0\pi^-}(m_*^2)}{\sqrt2}
\end{aligned}
$$

$$X^{\pi^0_u}_{\bar{B}^0\bar{K}^{*0}_0} =< \pi^0|(\bar{u}u)_L|0 >< \bar{K}^{*0}_0|(\bar{s}b)_L|\bar{B}^0 >= \frac{f_\pi}{\sqrt{2}}(m_B^2 - m_{K_0^*}^2)F_0^{\bar{B}^0\bar{K}_0^{*0}}(m_\pi^2) = \frac{f_\pi}{\sqrt{2}}(m_B^2 - m_{K_0^*}^2)r_{K\pi}F_0^{\bar{B}^0 a_0^+}(m_\pi^2)$$

$$X^{\bar{K}^{*0}_0}_{\bar{B}^0\pi^0} =< K^{*0}_0|(\bar{s}d)_L|0 >< \pi^0|(\bar{d}b)_L|\bar{B}^0 >= f_{K_0^{*0}}(m_B^2 - m_{\pi^0}^2)F_0^{\bar{B}^0\pi^0}(m_{K_0^*}^2) = f_{K_0^*}\left(m_B^2 - m_\pi^2\right)\frac{F_0^{B^0\pi^-}(m_*^2)}{\sqrt{2}}$$

$$\tilde{X}^{f_{0_d}}_{B^-\pi^-} =< f_0|\bar{d}d|0 >< \pi^-|\bar{d}b|B^- >= m_S\bar{f}^d_{f_0}\frac{m_B^2 - m_\pi^2}{m_b - m_d}F_0^{B^-\pi^-}(m_{f_0}^2) = -\frac{1}{\sqrt{2}}\frac{m_B^2 - m_\pi^2}{m_b - m_d}m_{f_0}\bar{f}^n_{f_0}F_0^{B^0\pi^-}(m_{f_0}^2)$$

$$\tilde{X}^{f_{0_s}}_{B^-K^-} =< f_0|\bar{s}s|0 >< K^-|\bar{s}b|B^- >= m_{f_0}\bar{f}^s_{f_0}\frac{m_B^2 - m_K^2}{m_b - m_s}F_0^{B^-K^-}(m_{f_0}^2) = \frac{m_B^2 - m_K^2}{m_b - m_s}r_{K\pi}m_{f_0}\bar{f}^s_{f_0}F_0^{B^0\bar{\pi}^-}(m_{f_0}^2)$$

$$\tilde{X}^{f_{0_s}}_{\bar{B}^0\bar{K}^0} =< f_0|\bar{s}s|0 >< \bar{K}^0|\bar{s}b|\bar{B}^0 >= m_{f_0}\bar{f}^s_{f_0}\frac{m_B^2 - m_K^2}{m_b - m_s}F_0^{\bar{B}^0\bar{K}^0}(m_{f_0}^2) = \tilde{X}^{f_{0_s}}_{B^-K^-} \tag{A.5}$$

for annihilation:

$$X^{B^-}_{f_0K^-} =< f^0K^-|(\bar{s}u)_L|0 >< 0|(\bar{u}b)_L|B^- >= -f_B(m_S^2 - m_K^2)F_0^{f^0K^-}(m_B^2)$$
$$X^{\bar{B}^0}_{\bar{K}^0f_0} =< \bar{K}^0f^0|(\bar{s}d)_L|0 >< 0|(\bar{d}b)_L|\bar{B}^0 >= -f_B(m_{f_0}^2 - m_K^2)F_0^{f_0\bar{K}^0}(m_B^2) = X^{B^-}_{K^-f_0}$$
$$X^{\bar{B}^0}_{K_0^{*-}\pi^+} =< K_0^{*-}\pi^+|(\bar{s}d)_L|0 >< 0|(\bar{d}b)_L|\bar{B}^0 >= -f_B(m_{K^*}^2 - m_\pi^2)F_0^{K_0^{*-}\pi^+}(m_B^2)$$
$$X^{B^-}_{\bar{K}_0^{*0}\pi^-} =< \bar{K}_0^{*0}\pi^-|(\bar{s}u)_L|0 >< 0|(\bar{u}b)_L|B^- >= -f_B(m_{K^*}^2 - m_\pi^2)F_0^{\bar{K}_0^{*0}\pi^-}(m_B^2)$$
$$X^{B^-}_{K_0^{*-}\pi^0} =< K_0^{*-}\pi^0|(\bar{s}u)_L|0 >< 0|(\bar{u}b)_L|B^- >= -f_B(m_{K^*}^2 - m_{\pi^0}^2)F_0^{K_0^{*-}\pi^0}(m_B^2)$$
$$X^{\bar{B}^0}_{\bar{K}_0^{*0}\pi^0} =< \bar{K}_0^{*0}\pi^0|(\bar{s}d)_L|0 >< 0|(\bar{d}b)_L|\bar{B}^0 >= -f_B(m_{K^*}^2 - m_\pi^2)F_0^{\bar{K}_0^{*0}\pi^0}(m_B^2) \tag{A.6}$$

with $r_{K\pi} = F^{BK}/F^{B\pi} \simeq f_K/f_\pi \simeq 1.21(9)$